Patna University

Courses of Study for M.A./M.Sc.(STATISICS)

SEMESTER SYSTEM

(Semesters I, II, III and IV)

<u>Semester – I</u>

Paper – 101

Real Analysis and Probability

Unit – I

Compact Set, Bolzano-Weirstrass theorem, Heine-Borel theorem, Uniform convergence, Power series & Radius of convergence

Unit – II

Classes of Sets, field, sigma-field, minimal sigma-field, Borel -field in R^k , Sequence of sets, limsup and liminf of sequence of sets, Measure, Probability measure, properties of a measure, catatheodory extension theorem (statement only), Lebesgue and lebesgue Stieltjes measure on R^k

Unit – III

Random variables, sequence of random variables, almost sure convergence. Convergence in probability (and in measure), Integration of a measurable function with respect to a measure, Monotone convergence theorem. Fatou's lemma, Dominated convergence theorem.

Unit – IV

Borel-Cantelli Lemma, Independence, Weak law and Strong law of large numbers for iid sequence

Unit – V

Convergence in distribution, characteristic function, uniqueness theorem, levy's continuity theorem (statement only), CLT for a sequence of independent random variables under Lindberg's condition, CLT for iid random variables.

Reference:

Apistol, T.M. : Mathematical Analysis, Narosa, Indian Ed.

Ash, Robert : Real Analysis and Probability, Academic Press

Billingsley, P : Probability and Measure, John – Wiley NY.

B.R. Bhat : Modern Probability theory, wiley Eastern Limited

H.L.Royden: Real Analysis, Prentice Hall of India.

Parzen E, : Modern Probability Theory and its Applications, Jonn wiley, 1960 New York. Shanti Narayan and Singhania M.D.R: Elements of Real Analysis, S. Chand & Company, New Delhi

Paper – 102, Linear Algebra

Unit – I

Vectors space, subspaces, linear dependence and independence, basis and dimension of a vector space, finite dimensional vector spaces, completion theorem, linear equations vector spaces with an inner product.

Unit – II

Linear transformations, algebra of matrices, row and column spaces of a matrix, elementary matrices, rank and inverse of a matrix, null space and nullity, partitioned matrices

Unit- III

Hermite canonical form, Idempotent matrices, solution of matrix equations

Unit – IV

Real quadratic form, reduction and classification of quadratic forms, index and signature, triangular reduction of a positive definite matrix.

Unit – V

Characteristic root and vectors, Cayley-Hamilton theorem, minimal polynomial, similar matrices, algebraic and geometric multiplicity of a characteristic root, reduction of a pair of real symmetric matrices, Hermiton matrices

Reference:

Biswas, S	: Topic in Algebra of Matrices, Academic Publication,
Hadley, G	: Linear Algebra, Narosa Publication house
Narayan,S	: A text Book of Matrices, S. Chand Publishing Company
Nashed, M (1976)	: Generalised Inverse and its Applications, Academic Press,
	New York
Searle, S.R. (1982)	: Matrix Algebra useful for Statistics, John Wiley and Sons.
Vasishtha A.R. and Vas	ishtha, A.K.: Matrices, Krishna Publication, Meerut

Paper 103

Sample survey and statistics for national development

Unit – I

Review of basic finite population sampling techniques Simple random sampling (wr/ wor), Stratified, Systematic sampling and related results on estimation of population mean/ total. Allocation problem in Stratified Sampling.

Unit- II

Unequal Probability Sampling: pps wr/wor methods including Lahri scheme and related estimators of finite population mean, Hansen- Hurwiytz and Desraj estimators for a general sample size and Murthy's estimators for a sample of size two, Ratio and regression estimators based on srswor method of sampling

Unit – III

Two stage sampling with equal number of second stage unit. Double sampling, Cluster sampling, Randomized response technique, Warner's model: related and unrelated questionnaire methods.

Unit – IV

Indices of development, Human Development index. Estimation of national income-product approach, income approach and expenditure approach. Population growth in developing and developed countries. Population projection using Leslie Matrix. Labour force projection

Unit – V

Measuring inequality in incomes, Gini's co-efficient, Theil's measure. Poverty measurementdifferent issues, measure of incidence and intensity, combined measure e.g. indices due to Kakwani Sen, etc.

Reference:

Chaubey, P.K. (1995): Poverty Measurement Issue, Approaches and Indices. New Age International (P) Ltd. New Delhi.

Chaubey, P.K. (2004): Inequality: Issues and Indices. Kanishka Publisher, Distributors New Delhi.

Chaudhari, A (2010) : Essentials of Survey sampling PHI learning Pvt. Ltd, New Delhi

Cochran, W.G. (1977): Sampling Techniques 3rd Edition, Wiley USA,

CSO (1980) : National Accounts Statistics – Sources and Health

Des Raj and Chandak (1998): Sampling theory, Narosa,

Keyfitz, N (1997) : Applied Mathematics Demography, Springer verlag

Murthy, M.N. (1967) : Sampling theory & methods, Statistical Publishing Society, Culcutta

Sen Amartya (2003) : Poverty and Famines, Oxford University Press, New Delhi

- Singh, D & Chaudhary F.S. (1996): Theory and Analysis of Sample Survey Designs, New Age International Publishers, New Delhi
- Sukhatme et al. (1984): Sampling theory of Surveys with Applications, Iowa State University Press & Indian Society of Agriculture Statistics, IASRI campus, Library Avenue, New Delhi – 110112

Mukhopadhyay, P (1997): Theory & Methods of Survey Sampling, Prentice Hall of India, New Delhi

UNDP Human Development Report 2011: Sustainability and Equity, A better Future for All UNDP Publication, USA

UNESCO – Principles of vital statistics system series M-12.

Paper – 104

Practical and group discussion

There will be one sitting of practical examination of three hours duration based on theory paper 101 to 103. The distribution of marks will be as follows

Practical note-book and viva-voice Group Discussion 35 marks 35marks

<u>Semester – II</u>

Distribution theory and Non- parametric tests

Unit – I

Brief review of basic distribution theory, Joint marginal and conditional p.m.fs and p.d.fs Standard discrete and continuous distributions, Bivariate normal & Bivariate exponential distribution

Unit – II

Function of random variables and their distributions using Jaicobian transformation and other methods. Compound, truncated and mixture distribution, Markov, Holder, Jensen and Liapounoff inequalities

Unit – III

Sampling distribution, Non-central Chi- square, t and F- distributions and their properties

Unit – IV

Order Statistics their distribution and properties. Joint and marginal distributions of order statistics. Distribution of extreme values and range and asymptotic distribution of median Empirical distribution function and its distributional properties, K-S goodness of fit.

Unit-V

Rank-test. One sample location problem, sign test and signed- rank-test, two sample K-S test, two sample location & scale problems.Wilcoxon_Mann-Whitney test, Non-parametric regression and analysis of variance techniques.

References:

Mood, A.M.; Graybill, F. A and Bose, D.C. : Introduction to theory of Statistics 3rd Edition.

Rohatgi, V.K. (2	984) : Introduction of Probability Theory and Mathematical Statistics	s,
	Wiley Eastern.	
Rao C.R. (1973)	: Linear Statistical Inference and its Application, Wiley Easte	rn

LINER MODELS AND REGRESSION ANALYSIS

Unit-I

Gauss-Markov Set-up, Normal equations and Least square estimates. Variance and Covariances of least square estimates, estimation of error variance, least square estimates with restrictions on parameters. Simultaneous estimates of linear parametric functions. Test of hypotheses for one and more than one linear parametric functions. Confidence intervals and regions.

Unit- II

Analysis of variance, multiple comparison tests due to Tukey and Scheffe, simultaneous confidence intervals.

Introduction to one-way random effect linear model and estimation of variance components

Unit- III

Simple linear regression, multiple regression fitting of polynomials and use of orthogonal polynomials.

Unit-IV

Residuals and their plots Tests for departure from assumptions such as fitness of the model, normality, homogeneity of variances and detection of outliers. Remedies.

Unit-V

Multicollinearity, Ridge regression and Principal Component regression, subset selection of explanatory variables, Introduction to logistic regression.

References:

Neter, I., Kutner, M.H. Nechtsheim, C.J. and Wasserman, W.(1996): Applied Liner Statistical Models WCB, Mcgraw Hill.

Rao, C.R. (1973) Linear Statistical Inference and its Application. Wiley Estern.

STATISTICAL COMPUTING

Unit –I

Programming in C++, Input/ Output Statements, Constant and Variables, data type, Variable's Scope, Control Statements, Arrays, Function and Pointer.

Unit-II

Concept of Object Oriented Programming, Class and object, Structure, Property of Inheritance, Polymorphism, Constructor and Destructor, Overloading and Overriding of functions, Static member variable and functions and Virtual functions in C++.

Unit-III

Elementary Java programming. Input/Output statement, Arrays, Function, Control Statements, Class and object. Properties of Inheritance, Overloading and Overriding of functions, Concept of Packages. Solutions of Statistical Problems based on C++ and JAVA.

Unit IV

MINITAB and SPSS for Graphics, Descriptive Statistics, Representation of Multivariate data, simple hypothesis tests, analysis of variance and linear regression.

Unit-V

Data base management, data warehousing and data mining.

References:

Bala Guruswany E: Programming in ANSI C++, Tata Mc Graw Hill

Dunham, M.H. : Data Mining Introduction and Advanced Topics, Dorling Kinerley (India) Pvt. Ltd.

Gottfried, Byson S. (1998) Progressing with C. Tata Mc Graw Hill Publishing Co. Ltd, New Delhi.

Guddis Tonny : Starting out with C++ Dream Tech, Press, New Delhi

Herbert. S. : Java TM 2, The Complete Reference, Tata Mc Graw Hill

PRACTICAL AND FIELD WORK

There will be one sitting of practical examination of three hours duration based on theory papers 201 to 203. The distribution of marks will be as follows

Practical note –book and viva voce	-	35 Marks
Report on field work	-	35 Marks

<u>Semester – III</u>

STATISTICAL INFERENCE

Unit -I

Sufficiency, Neyman Factorization criterion, Exponential families and Pitman families Invariance Property of Sufficiency ,Minimum Variance Unbaised estimators, completness, Lehmann-Scheffe Theorem.

Unit -II

BAN and CAN estimators. Pitman's method, Method of Scoring, Multinomial distribution with cell probabilities depending on a parameter, MLE in censored and truncated distributions.

UNIT-III

Neyman- Pearson Lemma and its applications. MP and UMP tests.UMP tests for simple null hypothesis against one sided alternatives and for one sided null against one sided alternatives in one parameter exponential family.

Unit-IV

Extension of these results to Pitman family when upper or lower end depends on the parameter and the distribution with MLR property. Liklihood Ratio-test. Asympotic distribution properties and application of LRT statistics.

Unit-V

Wald's SPRT, Determinantion of Constants, Approximate OC and ASN functions and their approximate expressions, OC and ASN fuction of Bernoulli. Normal and Exponential distributions. Variance stabilizing transformations. Asymptotic power of large sample tests.

References:

Casella, G and Berger, R.L	: Statistical Inference, Second Edition
Ferguson, T.S.(1967)	: Mathematical Statistics. Academic Press.
Kele, B.K.	: A First course on Parametric Inference, Second Edition
	Narosa
Lehmann,E.L.(1986)	: Theory of Point Estimation (Student Edition)
Lehmann,E.L.(1986)	: Testing Statistical Hypotheses (Student Edition)
Mood, A.M. Graybill, F.A	and Bose, D.C: Introduction to theory of Statistics, Mc Graw Hill
	International Edition
Rao, C.R.(1973)	:Linear Statistical Inference and its Application Wiley Eastern.

DESIGN AND ANALYSIS OF EXPERIMENTS

Unit-I

Introductuion to designed experiments. General Incomplete block design, Criteria for connectedness, balance and orthogonally Intrablock analysis of .BIBD-recovery of Interblock information. Youden Square design – intrablock analysis. Analysis of covariance.

Unit -II

Estimability, best point estimates/interval estimates of estimable linear parametric functions and testing of linear hypotheses.

Unit- III

Fixed, mixed and random effect models, Variance component's estimation. Study of various models.

Unit-IV

General Factorial experiments, study of 2 and 3 factorial experiments in randomized blocks. Complete and Partial confounding.

Unit-V

Application areas: Response surface experiments first order and orthogonal designs. Fractional replication for symmetric factorials, Split Plot design.

References:

Chakrabarti, M.C. (1962) Ma Pu	thematics of Design and Analysis of Experiments , Asia blication house, Bombay.
Das,M.N. and Giri,N.(1979)	: Design and Analysis of Experiments, Wiley Eastern
Dey A. (1986)	: Theory of Block Design, John Wiley & Sons.
Joshi,D.D.(1987)	: Linear Estimation and Design of Expeiments, Wiley Eastern
Montgomery, C.D. (1996)	: Design and analysis of Experiments, Wiley New York
Raghavarao, D. (1970), Con	struction and combinational Problems in Design of experiments,
Joh	n Wiley & Sons.

Elective Paper (Any one of the following)

1. Statistical Process and Quality Control

2.Reliability

STATISTICAL PROCESS AND QUALITY CONTROL

Unit-I

Basic concept of process monitoring and control, process capability and process optimization.

Unit-II

General theory and review of control charts for attribute and variable data, O.C. and A.R.L. of control charts ; control by gauging; Moving average and exponentially weighted moving average charts: Cu- um charts V- masks and decision intervals: Economic design of x bar chart.

Unit-III

Acceptance sampling plans for attributes; Single , Double and Sequential Sampling plans and their properties; plans for inspection by variables for one sided and type and Wald Wolfiwitz type and their properties, Bayesian sampling plans.

Unit-IV

Capability indices Cp.Cpk and Cpm; estimation, confidences, interval and tests of hypotheses relating to capability indices for normally distributed characteristics.

Unit-V

Use of Design of experiments in SPC; factorial experiments, fractional factorial designs, construction of such design and analysis of data. Multivariate quality control: Use of Control ellipsoid and of utility functions.

References:

Mitra, Amitava(2008)	:Fundamentals of Quality Control and Improvement. Prentice Hall
	of India . New Delhi.
Montogomery, D.C. (2009)	:Introduction to Statistical Quality Control : John Wiley & Sons
Montogomery, D.C. (1985)	:Design and analysis of Experiments: John Wiley & Sons
Ott,E.R.(1975)	:Process quality control; McGraw Hill
Wetherhill,G.B.(1977)	:Sampling Inspection and Quality Control; Halsted Press
Wetherhill, G.B. and Brown	n, D.W. : Statistical Process Control . Theory and practice; Chapman &
Hall.	

2. RELAIBILITY

Unit-I

Reliability, concept and measures; components and systems; coherent systems; reliability of coherent systems; cuts and paths; modular decomposition; bounds on system reliability; structural and reliability importance of components. Life distribution; reliability function; hazard rate; common life distributions-exponential, Weibull, gamma etc. Estimation of parameters and tests in these models.

Unit-II

Notions of ageing; IFR, IFRA, NBU, DMRL and NBUE Classes and their duals; closures of these classes under formation of coherent systems, convolutions and mixtures. Univariate shock models and life distribution arising out of them; bivariate shock models; common bivariate exponential distributions and their properties.

Unit-III

Reliability estimation based on failure times in variously complete and censored life tests and in tests with replacements of failed items; stress strength reliability and its estimation.

Unit-IV

Maintenance and replacement policies; availability of repairable systems; modelling of a repairable system by a non homogenous poisson process.

Unit-V

Reliablity growth models; probability plotting technique; Hollander-Proschan and Deshpande tests for exponentiality tests of HPP vs NHPP with repairable systems. Basic ideas of accelerated life testing .

References:

Reliability and life testing by S.K. Sinha, John Wiley Eastern Limited Reliability Engineering by L.S. Srinath, Affiliated East –West Press Pvt. Ltd. Stochastic Methods in Reliability theory by N. Ravichandran, Wiley Eastern Limit. Reliability Engineering by A.K. Govil, Tata McGraw- Hill Publication Company Limited

PRACTICAL

There will be two parts each of three hours duration based on the theory papers 301 to 303. The distribution of marks will be as follows :

Part 1

28 marks

Part 2

28 marks

Practical Note Book And Viva – Voce 14 marks

<u>Semester – IV</u>

STOCHASTIC PROCESSES

Unit – I

Introduction to stochastic processes: Classification of stochastic processes according to state space and time domain, Continuous state Markov Chain, Chapman- Kolmogorov equations Calculation of n-step transition probability and its limit.

Unit – II

Stationary distribution, classification of states, Markov - chain.Random walk and Grabler's ruin problem, applications to social, biological and physical science.

Unit- III

Discrete state space, continuous time, MC.Kolmogrov –Feller differential equations Poisson process. Birth and Death process.

Unit- IV

Applications to queues and storage problems, Wiener process as a limit of random walk, first passage time and other problems.

Unit-V

Renewal theory: elementary renewal theorem and application. Statement and uses of Key renewal theorem, study of residual life time process. Branching process: Galton-Watson branching process, probability of ultimate extinction, distribution of population size, Introduction to martingale.

References:

B.R. Bhatt : Stochastic Models, New Age Publisher

Singh, BM : Measure, Probability and Stochastic Processes, South Asian Publisher, New Delhi.

Seldon N Ross : Stochastic Processes, Wiley Student Publication

S.K Srinivasan and K.M Mehta: Stochastic Processes, Tata Macgraw Hill Publicing company Feller, W. (1968) : Introduction of Probability and its Applications, Vol.1 Wiley Eastern Karlin'S and Taylor, H.M. (1975) : A first Course in Stochastic Processes, Vol.1 Academic Press.

Medhi, J (1998)	: Stochastic Processor, Third Edition, New age International (P) Ltd.
Publication.	
Parzen, E. (1962)	: Stochastic Processes, Holden-Day.
Ross, S.M (1983)	Stochastic Process. John Wiley & Sons.

Paper 402 Multivariate Analysis

Unit – I

Multivariate normal distribution, its properties and characterizations, Random sampling from multivariate normal distribution. Maximum likelihood estimators of parameters. Distribution of sample mean vector.

Unit – II

Wishart distribution- its derivation and properties. Distributions of sample generalized variance. Distribution of quadrative form

Unit – III

Null and Non-Null distribution of simple correlation co-efficient. Null distribution of partial and multiple correlation coefficient. Distribution of sample regression coefficients.

Unit – IV

Hotelling's T^2 statistics- its distribution and properties, Applications in test on mean vector for one and more multivariate normal polpulations, Mahalnobis D^2 classification and discrimination procedures for discrimination between two multivariate normal populations, Fisher's Discriminant function tests associated with discriminant functions.

Unit – V

Canonial variates and canonical correlation. Principal Component Analysis. Elements of factor analysis and cluster analysis.

References :

Anderson, T.W. (2003) : An introduction to multivariate statistical analysis 3^{rd} Ed. John Wiley & sons

Johnson, R.A. & Wichern, D.W. (2001) : Applied Multivariate Statistical Analysis 3rd Ed. Pretice- Hall of India, New Delhi.

Kshirsager, A.M. (1972): Multivariate Analysis, Marcel Dekker

Morrison, D.F. (1976) : Multivariate Statistical Analysis 2nd Ed. Mc Graw Hill

Rao, C.R (1973) : Linear Statistical Inference and its application 2^{nd} Ed. Wiley

Paper – 403 Elective Paper (Any one of the Following)

- 1. Operation Research
- 2. Demography
- 3. Actuarial Statistics

Paper – 404 Practical & Dissertation Dissertation + Viva Voce

There will be one sitting of practical examination of three hours duration based of three papers 401-403.

Practical	35 marks
Practical note book + Viva Voce	10 marks
Dissertation & Viva Voce	35 marks

OPERATIONS RESEARCH

Unit – I

Definition and scope of operation research: Phases in Operation Research: models and their solution; decision-making under uncertainty and risk, use of different criteria: sensitivity analysis

Unit – II

Programming : Linear Programming and Non linear Programming. Review of LP problems, duality theorem transportations and assignment problems; Non-linear programming- Kuhn Tuckey conditions. Wolfe's and Beals's algorithms for solving quadratic programming problem. Bellman's principle of optimality, general formulation computation methods and application of dynamic programming.

Unit – III

Simulation, Analytical structure of inventory problems; EOQ formula of Harris, its sensitivity analysis and extension allowing quantity discounts and shortages. Multi –item inventory subject to constraints. Models with random demand, the static risk model.

Unit – IV

Queuing models- specifications and effectiveness measure. Steady –state solution of M/M/I and M/M/C models with associated distribution of queue-length and waiting time. M/G/I queue and Pollazcek Khinchine result. Steady-state solutions of $M/E_K/1$ queues. Machine interference problems.

Unit – V

Sequencing and Scheduling problem. 2 machine n-jobs and 3-machine n jobs with identical machine sequence for all jobs; 2 jobs n machine problem. Branch and bound method for solving travelling salesman problems

References:

Chaurchman, C.W, Ackoff R.L. and Arnoff E.L. (1975) : Introduction to Operation Research: John Wiley

Hillier F.S. and Leiberman G.J. (1962) : Introduction to Operation Research; Holden Day

Kanti Swarup, Gupta, P.K and Singh M.M (1985) : Operations Research; Sultan Chand & Sons

Philips D.T, Ravindran A and Solberg J. : Operation & Research, Principle and Practice Taha H.A. (1982) : An introduction of Operation Research; Macmillan

Demography

Unit –I

Coverage and Content errors in demographic data. Use of balancing equation and equations and Chandrasekharan - Deming formula to check completeness of registration data. Adjustment of age data. Use of Whipple, Myer and UN indices. Population composition dependency ratio.

Unit – II

Measures of fertility, Stochastic models for reproduction, distribution of time to first birth, inter-live birth interval and of number of births (for both homogeneous and non-homogeneous groups of women) estimation of parameters; estimation of parity progression ratios from open birth interval data.

Unit – III

Measure of Mortality; construction of abridged life tables, Distribution of life table functions and their estimation.

Unit – IV

Stable and quasi –stable populations, intrinsic growth rate. Models for Population growth and their fitting to population data. Stochastic models for Population growth.

Unit – V

Stochastic models for migration and for social and occupational mobility based on Markov chains. Estimation of measure of mobility. Methods for population projection. Use of Lesile method.

Reference :

Benjamin B. (1969) : Demographic Analysis, Geogre Allen and Unwin

Cox, P.R. (1970) : Demography, Cambridge University Press

Henery. S and Siegel J.S and Associate (1971) : The method and materials of Demography; Washington D.C. US Bureau of Census.

Nathan K. (1968) : Introduction to the Mathematics of Population Reading, Mann : Addison-Wesley

Ram Kumar : Technical Demography

Keyfit, N (1970) : Applied Mathematical Demography,; Springer Verlag

ACTUARIAL STATISTICS

Unit -I

Utility theory, insurance and utility theory, models for individual claim and their sums, survival function, curate future life time, force of morality.

Life table and its relation with survival function, example, assumptions for fractional ages, some analytic laws of mortality, select and ultimate tables.

Unit -II

Distribution of aggregate claims, compound Poisson distribution and its applications. Distribution of aggregate claims.

Principles of Compound Interest: nominal and effective rates of interest and discount, force of interest and discount, compound interest, accumulation factor, continuous compounding.

Unit – III

Life insurance : Insurance payable at the moment of death and at the end of the year of death , endowment insurance , deferred insurance and varying annuities , recursions , complete annuities-immediate and apportion able annuities due.

Unit – IV

Net premiums : Continuous and discrete premiums, true monthly payment premiums, apportion able premiums, commutation functions, accumulation type benefits.

Payment premium , apportion able premiums, commutation functions, accumulation type benefits.

Unit – V

Net Premium reserves : Continuous and discrete net premium reserve , reserves on a semicontinuos basis, reserves based on true monthly premium, reserves on an apportion able or discounted continuous basis , reserves at fractional durations.

Claim amount distributions, approximating the individual model, stop loss insurance.

References :

- 1. A first course in probability, 6th Ed. New Jersey Prentice Hall, India
- 2. Actuarial Mathematics by N.L. Bower, Gerber, Hickman, D.A. Jones, Nesbit, The society of Actuaries
- 3. Actuarial Statistics by S.R. Deshmukh, University Press
- 4. Actuarial Theory by W.A. Robertson and F.A. Ross, Oliver and Boyd, London.
- 5. Economics by John Sloman, Pearson Education (LPE)
- 6. Life Insurance Mathematics by Hans U. Gerber, Springer
- 7. Technical Demography by R. Ram Kumar, New Age International Publishers.